Structural contact problems for large deformations - Comparison of steady and non steady normal field

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We will present a comparison between two formulations of the normal vector field for contact algorithms based on the mortar method. First the non steady normal field is discussed. The non steadiness is a result of the C^0 continuity of the boundary discretization. This is the common result if one discretize the domain with classical finite element methods. Second we will present results for a special normal field distribution. We average the nodal normal vector of two ascending edges and interpolate this nodal normal throughout the edges. We have implemented both methods and present comparisons based on numerical experiments.

1 Normal vectors

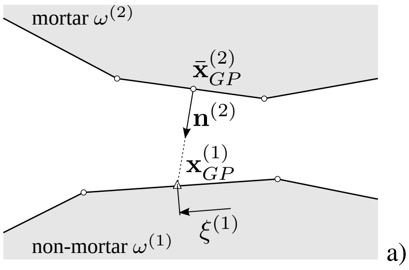
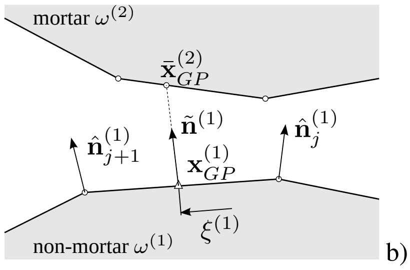


Fig. 1 Discontinuous mortar side normal vector (**a**) and continuously averaged non-mortar side normal vector (**b**).



Discontinuous mortar side normal vectors (Figure 1a) The proposed mortar side normal was presented in [1] and also in other publications. As the normal vector is normal to its adjacent edge on the mortar side it is obvious that the direction of the normal vector jumps on start and end nodes of a mortar edge.

Continuously averaged non-mortar side normal vectors (Figure 1b) Yang presented in his paper [3] (see also [4], [5], [6]) a new method on defining the normal vectors. Those methods have in common that there is an averaged unit normal defined on start and end nodes of the contact edges (mostly non-mortar side edges). This averaged normal is then interpolated over the edges and therefore continuity is recovered.

2 Problems related to C^0 only surfaces in contact mechanics

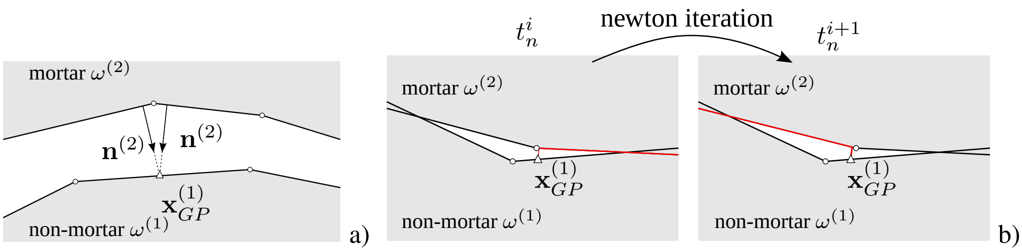


Fig. 2 Uniqueness of projection (**a**) and oscillation behavior of the nearest point projection (**b**).

Uniqueness of projection As discussed and shown in [2] there are several difficulties for solvability and uniqueness of the closest point projection. One of those issues is shown in figure 2a. For concave mortar surfaces two possible normal vectors are defined. This issue can be solved by averaging. The problem on boundaries corners and others in [2] discussed problems remain.

Oscillation of projection For the nearest point projection we have to determine the corresponding edge for a given projection point. This edge might change during the Newton-Raphson procedure from one iteration to the next. This situation is shown in figure 2b. If the "best" projection edge changes back again in the next iteration the projection starts to oscillate and no convergence can be reached. This problem is *not influenced* by averaging.

Smoothing of contact traction The last motivation we could consider for doing the averaging is smoothing of contact traction. As the traction is represented by the scalar Lagrange value inside the mortar method the normal vector gives the direction. If the normal vector direction jumps - the contact traction jumps too. This might decrease the quality of solution and is further investigated in the numerical examples.

3 Numerical Experiments

We have used the so called "ironing problem". In figure 3 you can see the dimensions of the problem. It consists of a block and a slab. We are moving the block downwards $\Delta v = 15[L]$ (highlighted in red) and then move it in horizontal direction over the slab $\Delta u = 260[L]$ (highlighted in green). We used a neo-Hookean material with the parameters $E_{\text{slab}} = 1.0e3\ [F/L^2]; \nu_{\text{slab}} = 0.3; E_{\text{block}} = 1.0e4\ [F/L^2]; \nu_{\text{block}} = 0.3$. To see the influence of the discretization level two different meshes where calculated - a coarse mesh (see figure 4) and a fine mesh (see figure 5).

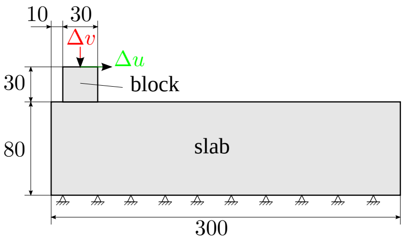


Fig. 3 Ironing problem

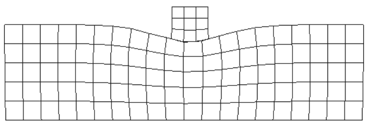


Fig. 4 Coarse mesh

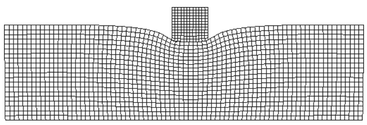


Fig. 5 Fine mesh

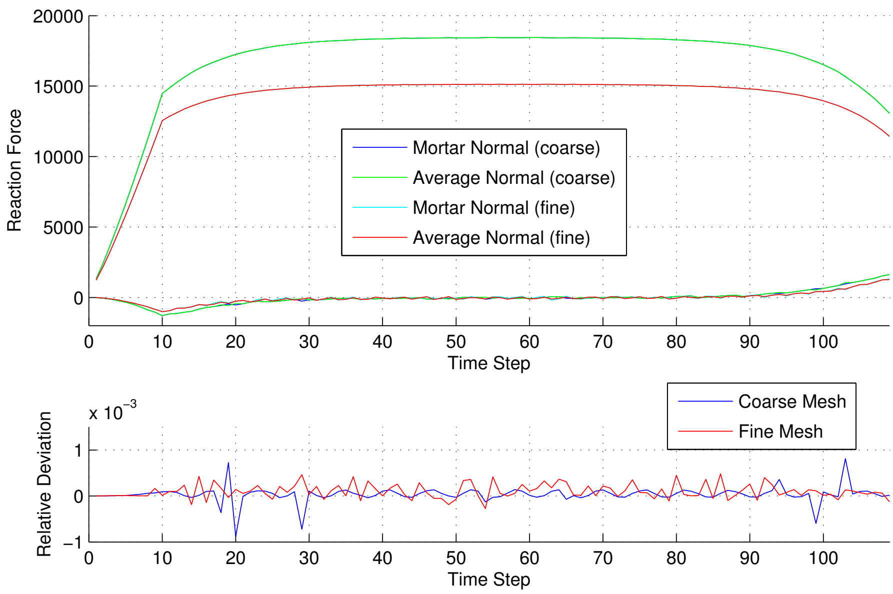


Fig. 6 Continuously averaged non-mortar side normal vector

On top of Figure 6 one can see the results for the reaction force on top of the block. The green line is the resulting vertical reaction force for the coarse mesh and averaged normal field. Indeed this should be 2 lines - the blue line for the mortar normal is covered by the green one. The red line is the resulting vertical reaction force for the fine mesh and averaged normal field. One can see that the coarse mesh is stiffer then the fine one but still the differences between averaged normal and mortar normal are to small to distinguish in this plot. To complete the plot the horizontal force is shown too and similar properties can be seen.

On the bottom of figure 6 the relative deviation between the averaged and the mortar side normal for the coarse and the fine mesh are plotted. We would like to point the readers attention to the exponent of -3. So the differences are negligible.

4 Conclusions

We showed that for our experiments the influence of normal averaging on the robustness of the algorithm and the quality of solution is negligible. Therefore we came to the conclusion that the influence of the only C^0 steady geometry is more dominant then the influence of the non steady normal field.

References

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