Structural contact problems for large deformations - Comparison of steady and non steady normal field

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Outline

Classification and Motivation

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Continuous Contact Kinematics

Discrete Contact Kinematics

Outline

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Discrete Contact Kinematics

Integration Scheme

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What do we focus on in this presentation?

- Only a very small subset of the whole problem

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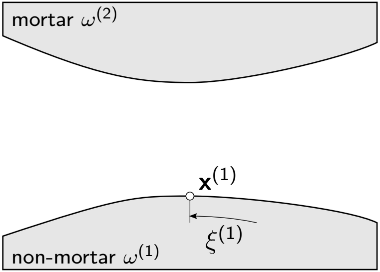
What do we focus on in this presentation?

- Only a very small subset of the whole problem
- Inconsistencies of the kinematics due to discretization
- Analyze the advantages of an averaged (steady) normal field

Continuous Contact Kinematics



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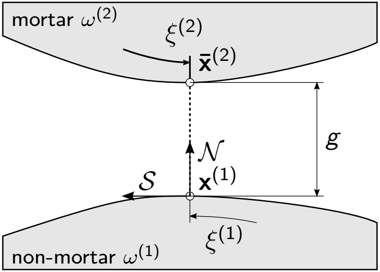

Identification $\bar{\mathbf{x}}^{(2)} = \mathbf{x}^{(2)}\left(\xi^{(2)}(\xi^{(1)})\right)$ defined by closest point projection

$$\begin{aligned} \frac{\partial}{\partial \xi^{(2)}} \left[g(\xi^{(2)}, t) \right] \Bigg|_{\xi^{(1)}=\text{const}} &= 0 \\ \left\{ \bar{\mathbf{x}}^{(2)} - \mathbf{x}^{(1)} \right\} \cdot \mathcal{S} &= 0 \end{aligned}$$

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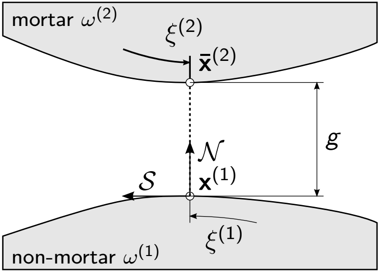

$$\bar{\mathbf{x}}^{(2)} - \mathbf{x}^{(1)} = g\,\mathcal{N}$$

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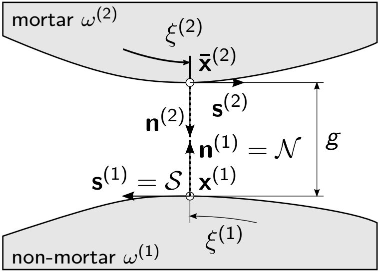

$$\bar{\mathbf{x}}^{(2)} - \mathbf{x}^{(1)} = g\,\mathbf{n}^{(1)}$$

Identification $\bar{\mathbf{x}}^{(2)} = \mathbf{x}^{(2)}\left(\xi^{(2)}(\xi^{(1)})\right)$ defined by closest point projection

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$$\left\{\bar{\mathbf{x}}^{(2)} - \mathbf{x}^{(1)}\right\} \cdot \mathbf{s}^{(1)} = 0$$

In the continuous case $\mathcal{S} = \pm\,\mathbf{s}^{(1)} = \mp\,\bar{\mathbf{s}}^{(2)}$

Discrete Contact Kinematics

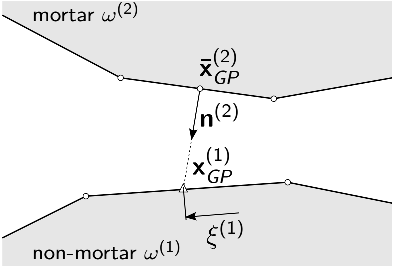
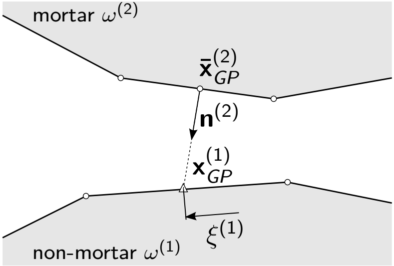


Figure: Mortar side normal with jump on mortar nodes

Discrete Contact Kinematics



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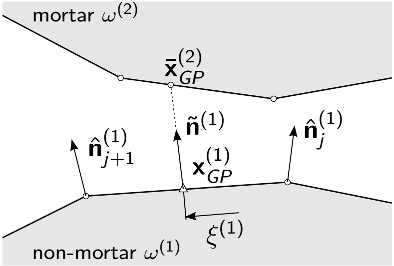
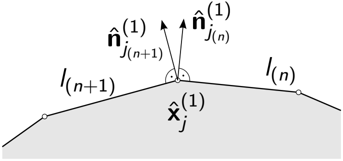


Figure: Continuous averaged non mortar side normal

Averaging method

Yang, Laursen, et al. IJNME 2005 (**62**)

$$\hat{\tilde{\mathbf{n}}}_j^{(1)} = \frac{l_{(n+1)}\hat{\mathbf{n}}_{j_{(n)}}^{(1)} + l_{(n)}\hat{\mathbf{n}}_{j_{(n+1)}}^{(1)}}{\| l_{(n+1)}\hat{\mathbf{n}}_{j_{(n)}}^{(1)} + l_{(n)}\hat{\mathbf{n}}_{j_{(n+1)}}^{(1)} \|}$$



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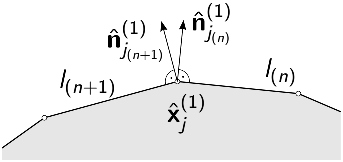
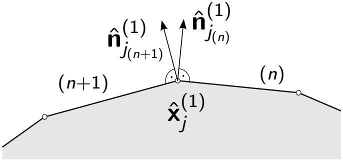


Popp et al. IJNME 2009 (**79**)

$$\hat{\tilde{\mathbf{n}}}_j^{(1)} = \frac{\hat{\mathbf{n}}_{j_{(n)}}^{(1)} + \hat{\mathbf{n}}_{j_{(n+1)}}^{(1)}}{\left\| \hat{\mathbf{n}}_{j_{(n)}}^{(1)} + \hat{\mathbf{n}}_{j_{(n+1)}}^{(1)} \right\|}$$



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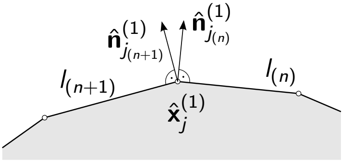

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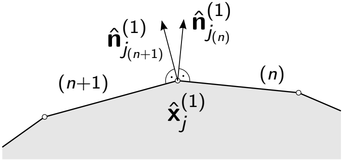
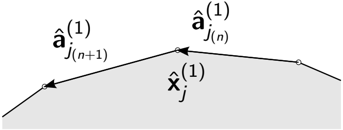


Tur, Wriggers CMAME 2009 (**198**)

$$\hat{\tilde{\mathbf{n}}}_j^{(1)} = \frac{\hat{\mathbf{a}}_{j_{(n)}}^{(1)} + \hat{\mathbf{a}}_{j_{(n+1)}}^{(1)}}{\left\| \hat{\mathbf{a}}_{j_{(n)}}^{(1)} + \hat{\mathbf{a}}_{j_{(n+1)}}^{(1)} \right\|} \times \mathbf{e}_3$$



Integration Scheme

It's common to integrate on the non-mortar surface

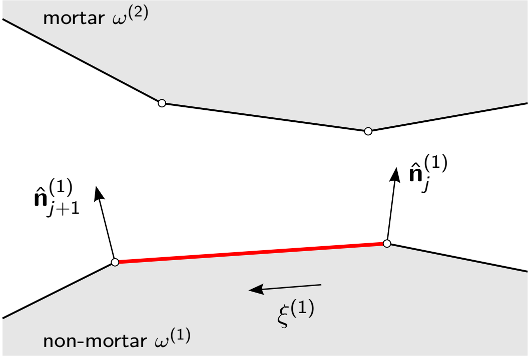


Figure: Integration on $\gamma_C{}^{(1)}$

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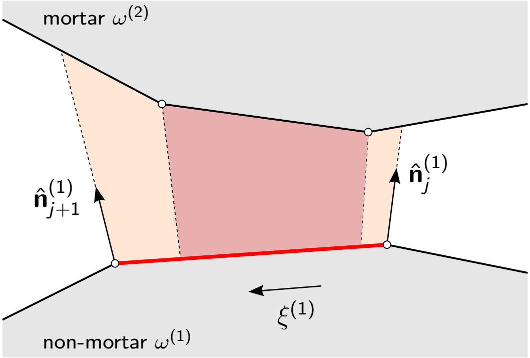

Figure: Integration through segmentation (see [Laursen et al.])

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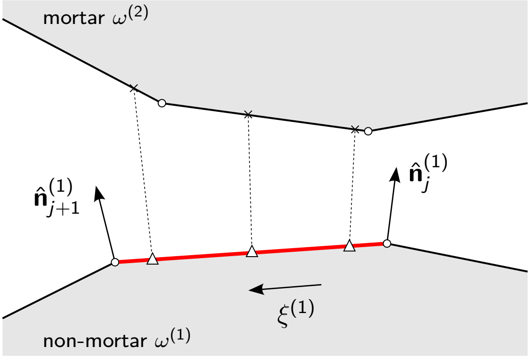
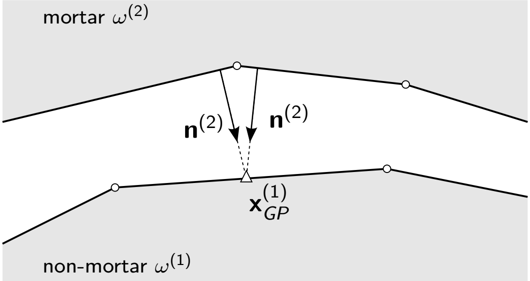


Figure: Concentrated integration (see [Wriggers et al.])

Related Issues

Uniqueness of projection (especially on concave mortar surfaces):

Konyukhov Comput. Mech., 2003, (**33**)



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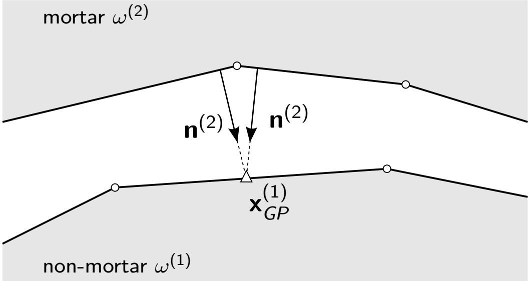
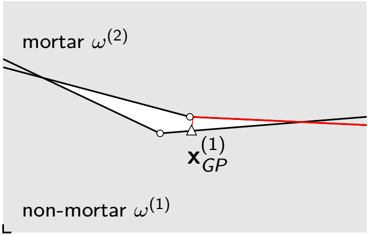


Solution: Eliminated by averaged normals (recovers C^1 continuity).

Related Issues

Oscillation of projection edge



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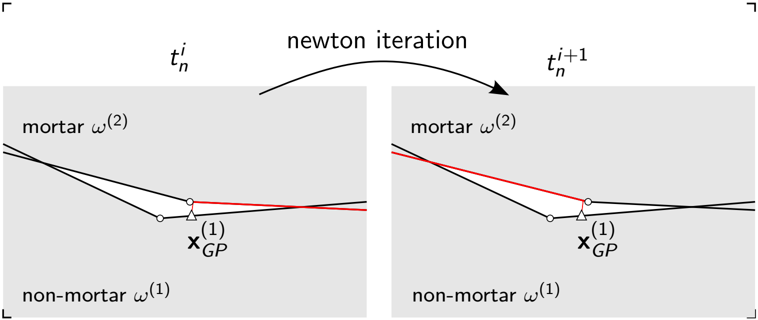

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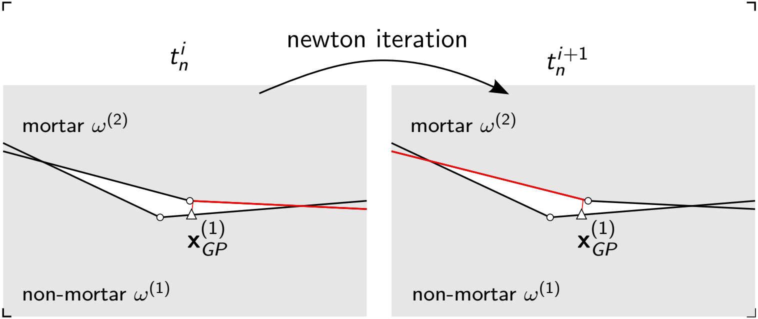

Solution: Averaging doesn't fix this problem. Convergence can be recovered by holding the projection edge fixed.

Related Issues

Smoothing of traction

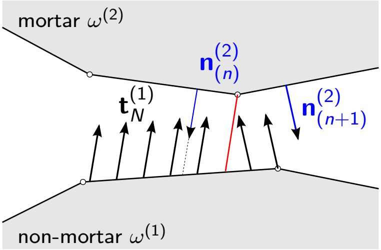

Figure: Mortar side normal with jump on mortar nodes leads to non continuous traction

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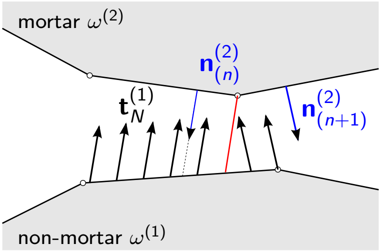

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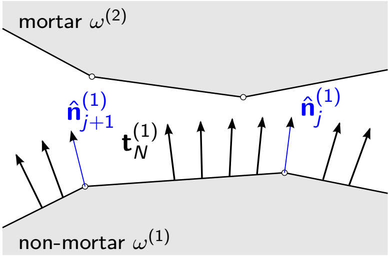
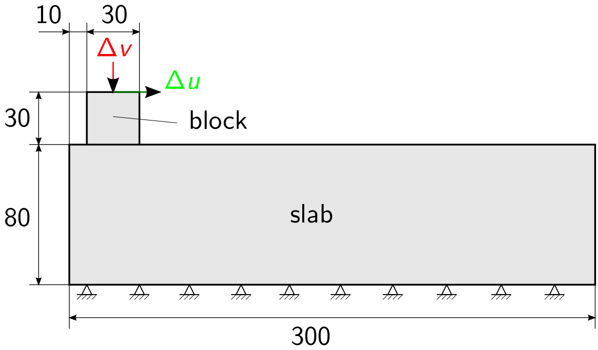


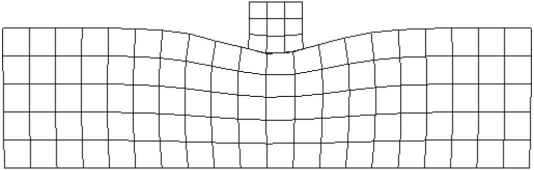
Figure: Continuously averaged non mortar side normal leads to continuous traction

Ironing Problem



$$E_{\text{slab}} = 1.0e3\ [F/L^2]$$
$$\nu_{\text{slab}} = 0.3$$
$$E_{\text{block}} = 1.0e4\ [F/L^2]$$
$$\nu_{\text{block}} = 0.3$$
$$\Delta u = 260.0\ [L]$$
$$\Delta v = 15.0\ [L]$$

Ironing Problem



coarse mesh

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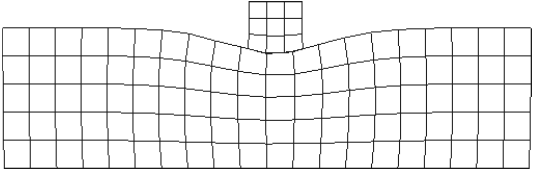

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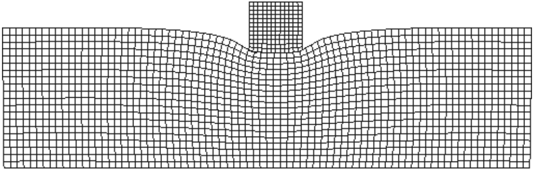
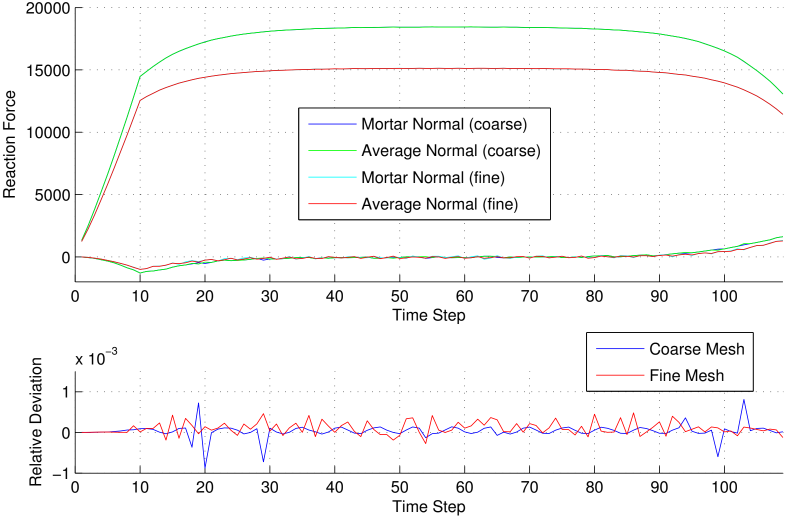
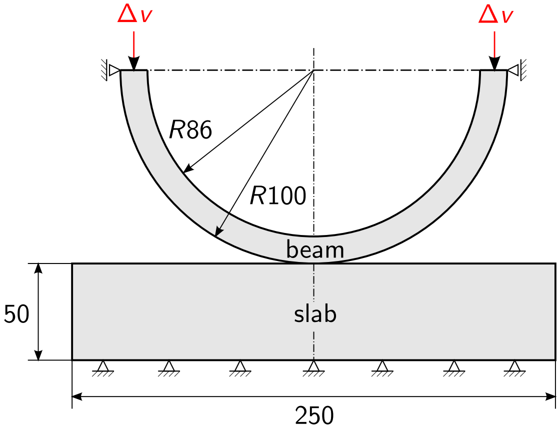


fine mesh

Ironing Problem



Curved Beam



$$E_{\text{slab}} = 300\ [F/L^2]$$
$$\nu_{\text{slab}} = 0.3$$
$$E_{\text{beam}} = 1.0e4\ [F/L^2]$$
$$\nu_{\text{beam}} = 0.3$$
$$\Delta v = 70.0\ [L]$$

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What are the results?

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- The effort of linearization and computation time rises through averaging.

In our experiments the influence of averaging is negligible.

To admit: The numerical experiments are based on “concentrated” integration methods (see Wriggers et al.). The influence through a segmented integration method should be further investigated.