Mortar-Based Contact Formulation for Large Deformations

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Outline

Verbal problem definition

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Continuous contact problem

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Finite element formulation and discretization issues

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Conclusion

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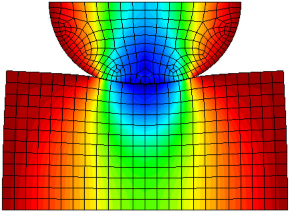
- Contact of two non rigid structural domains



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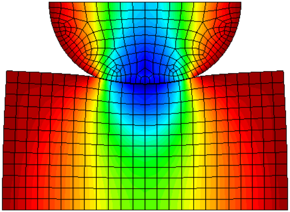
- Contact of two non rigid structural domains
- Arbitrarily formed contact surface



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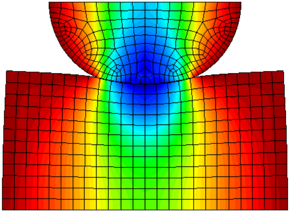

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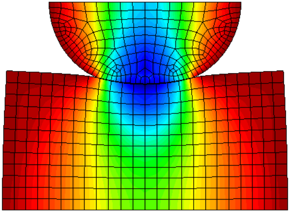

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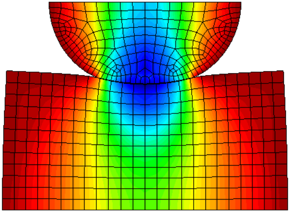

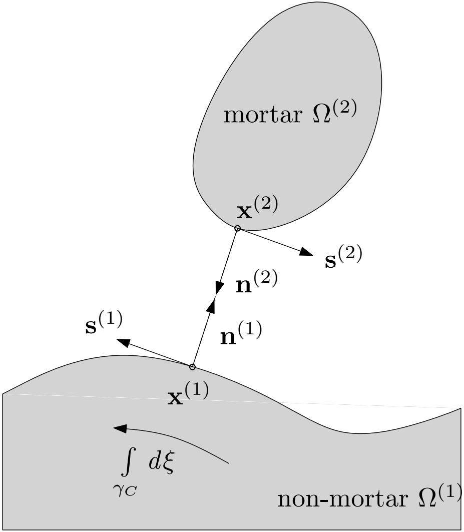

Figure: Continuous contact kinematics

Boundary Value Problem

$$\begin{aligned} div(\mathbf{F}^{(i)}\mathbf{S}^{(i)}) + \hat{\mathbf{b}}_0^{(i)} &= 0 && \text{in} && \Omega^{(i)} \\ \mathbf{u}^{(i)} &= \hat{\mathbf{u}}^{(i)} && \text{on} && \Gamma_D^{(i)} \\ \mathbf{P}^{(i)}\mathbf{N}^{(i)} &= \hat{\mathbf{t}}_0^{(i)} && \text{on} && \Gamma_N^{(i)} \end{aligned}$$

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\end{aligned}
$$

Contact traction $\mathbf{t}_C^{(i)}$ on γ_C (split formulation) in current configuration:

$$
\mathbf{t}_C^{(i)} = t_N^{(i)}\mathbf{n}^{(i)} + t_T^{(i)}\mathbf{s}^{(i)} \quad t_N^{(i)} = \mathbf{t}_C^{(i)} \cdot \mathbf{n}^{(i)} \quad t_T^{(i)} = \mathbf{t}_C^{(i)} \cdot \mathbf{s}^{(i)}
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Karush-Kuhn-Tucker (KKT) conditions of normal contact:

$$\begin{aligned} g(\mathbf{X}, t) &\geq 0 \\ t_N^{(1)} &\leq 0 \\ t_N^{(1)} g(\mathbf{X}, t) &= 0 \end{aligned}$$

Variational Formulation

Weak formulation of BVP:

$$\delta\Pi(\mathbf{u},\delta\mathbf{u}) = \sum_{i=1}^{2}\left\{\int_{\Omega^{(i)}}\left[\delta\mathbf{E}^{(i)} : \mathbf{S}^{(i)} - \hat{\mathbf{b}}_0^{(i)}\cdot\delta\mathbf{u}^{(i)}\right]\mathrm{d}\Omega - \int_{\Gamma_N^{(i)}}\hat{\mathbf{t}}_0^{(i)}\cdot\delta\mathbf{u}^{(i)}\mathrm{d}\Gamma\right\}$$

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Variant 1: (see [Puso, Laursen et al.])

$$\delta\Pi(\mathbf{u},\delta\mathbf{u}) = \delta\Pi_{int}(\mathbf{u},\delta\mathbf{u}) \underbrace{- \sum_{i=1}^{2}\left[\int_{\Gamma_N^{(i)}} \hat{\mathbf{t}}_0^{(i)} \cdot \delta\mathbf{u}^{(i)} \mathrm{d}\Gamma\right]}_{\delta\tilde{\Pi}_{ext}(\mathbf{u},\delta\mathbf{u})} \underbrace{- \sum_{i=1}^{2}\left[\int_{\Gamma_C^{(i)}} \mathbf{t}_{C,0}^{(i)} \cdot \delta\mathbf{u}^{(i)} \mathrm{d}\Gamma\right]}_{\delta\Pi_C(\mathbf{u},\delta\mathbf{u})}$$

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and for equilibrium

$$\mathbf{t}_C^{(1)}\mathrm{d}\gamma_C^{(1)} = -\mathbf{t}_C^{(2)}\mathrm{d}\gamma_C^{(2)} \quad \text{with} \quad \gamma_C^{(1)} = \gamma_C^{(2)} = \gamma_C$$

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on current configuration and using balance of linear momentum

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further a weak form of the non-penetration condition is needed

$$0 \leq \int_{\gamma_C^{(i)}} \delta t_N^{(1)}\, g(\mathbf{X},t) \mathrm{d}\gamma_C$$

Variant 2: (see [Fischer, Wriggers])

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we obtain

$$\begin{aligned} \delta\tilde{\Pi}_C(\mathbf{u}, \delta\mathbf{u}) &= \underbrace{\int_{\gamma_C} t_N^{(1)} \left(\delta\mathbf{u}^{(1)} - \delta\mathbf{u}^{(2)}\right) \cdot \mathbf{n}^{(1)} \mathrm{d}\gamma_C}_{\delta\Pi_C(\mathbf{u},\delta\mathbf{u}) \text{ already for split formulation}} \\ 0 &\leq \underbrace{\int_{\gamma_C} \delta t_N^{(1)}\, g(\mathbf{X}, t)\mathrm{d}\gamma_C}_{\text{weak KKT}} \end{aligned}$$

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Contact pressure t_N

- Penalty Method: $t_N^{(1)} = \varepsilon g(\mathbf{X}, t) \quad \Rightarrow \quad \tilde{\Pi}_C = \frac{1}{2} \int_{\Gamma_C} \varepsilon \, g(\mathbf{X}, t)^2 \mathrm{d}\Gamma$

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- Augmented Lagrange Method (see [Laursen et al.])

Discretization

We interpolate geometry and displacement field of slave and master surface

$$\mathbf{x}^{(i)} = \sum_{k=1}^{n_{(i)}} N_k(\xi^{(i)}(\mathbf{X}^{(i)})\mathbf{x}_k^{(i)}$$

$$\mathbf{u}^{(i)} = \sum_{k=1}^{n_{(i)}} N_k(\xi^{(i)}(\mathbf{X}^{(i)})\mathbf{d}_k^{(i)}$$

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also the Lagrange multiplier with dual shape functions

$$\boldsymbol{\lambda} = \sum_{j=1}^{n_{(1)}} \Phi_j(\mathbf{X}^{(1)})\mathbf{z}_j$$

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constructed with biorthogonality condition

$$\int_{\gamma_C^h} \Phi_j^{(1)}(\xi^{(1)}) N_k^{(1)}(\xi^{(1)}) \mathrm{d}\gamma_C = \delta_{jk} \int_{\gamma_C^h} N_k^{(1)}(\xi^{(1)}) \mathrm{d}\gamma_C$$

Non steady normal field

In continuous spaces $\mathbf{n}^{(1)} = -\mathbf{n}^{(2)}$ but not for discrete case

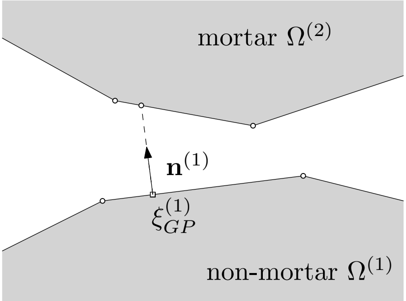

Figure: Non-mortar normal

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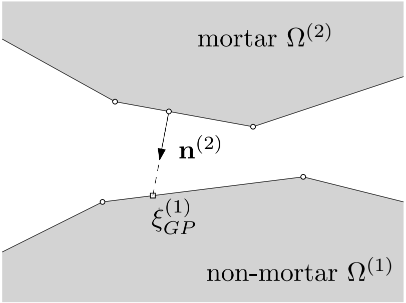

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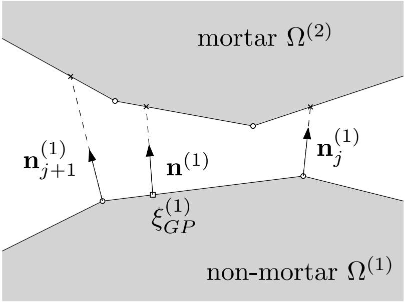

Figure: Average normal

$$\mathbf{n}_j^{(1)} = \frac{\mathbf{n}_{j1}^{(1)} + \mathbf{n}_{j2}^{(1)}}{\|\mathbf{n}_{j1}^{(1)} + \mathbf{n}_{j2}^{(1)}\|}$$

$$\mathbf{n}^{(1)} = \sum_{j=1}^{n_{(1)}} N_j(\xi^{(1)})\mathbf{n}_j^{(1)}$$

Numerical integration

It's common to integrate on the non-mortar surface

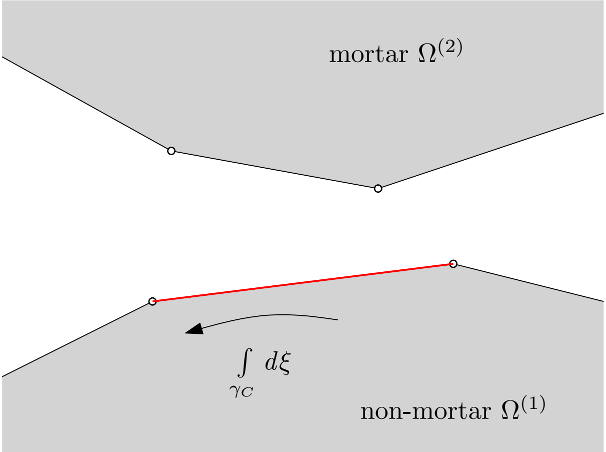

Figure: Integration on $\gamma_C^{(1)}$

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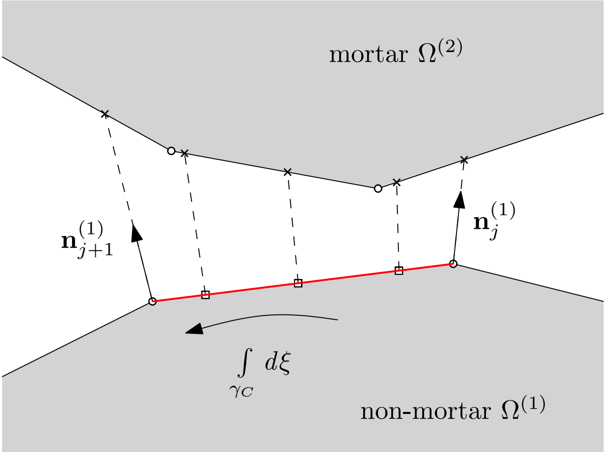

Figure: Concentrated integration (see [Wriggers et al.])

Numerical integration

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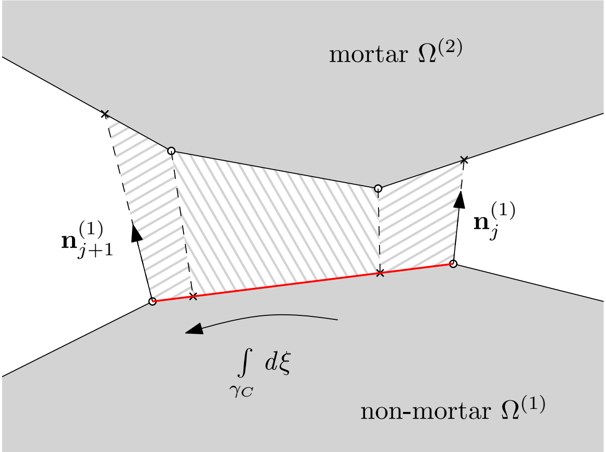

Figure: Integration by clipping (see [Laursen et al.])

Active Set Strategy

Let's define an "averaged nodal gap" with the help of the weak KKT

$$-\int_{\gamma_C} \delta\lambda_N \, g(\mathbf{X}, t) \mathrm{d}\gamma_C \quad \geq \quad 0$$

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\begin{aligned}
-\int_{\gamma_C} \delta\lambda_N \, g(\mathbf{X}, t) \mathrm{d}\gamma_C &\geq 0 \\
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in discrete form:

$$\tilde{g}_j = \mathbf{n}^{(1)} \cdot \left[\int_{\gamma_c^h} \Phi_j N_I^{(2)} \mathrm{d}\gamma_C^h \mathbf{x}_I^{(2)} - \int_{\gamma_c^h} \Phi_j N_I^{(1)} \mathrm{d}\gamma_C^h \mathbf{x}_I^{(1)} \right]$$

Active Set Strategy

The discrete form of the KKT

$$
\begin{aligned}
\tilde{g}_j &\geq 0 \\
\left(z_n\right)_j &\geq 0 \\
\left(z_n\right)_j \tilde{g}_j &= 0
\end{aligned}
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Fixed point Newton-Raphson algorithm (see [Fischer et al.])

$$\tilde{p}_j = -\frac{\int_{\gamma_C{}^h} N_j^{(1)} \Phi_k \, \mathrm{d}\gamma_C^h \, (\lambda_N)_k}{\int_{\gamma_C{}^h} N_j^{(1)} \, \mathrm{d}\gamma_C^h}$$

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After reaching convergence in Newton-Raphson check

- For active nodes: $\tilde{p}_j < 0 \Rightarrow$ Deactivate node
- For inactive nodes: $\tilde{g}_j < 0 \Rightarrow$ Activate node

Active Set Strategy

The discrete form of the KKT

$$\begin{aligned} \tilde{g}_j &\geq 0 \\ (z_n)_j &\geq 0 \\ (z_n)_j\, \tilde{g}_j &= 0 \end{aligned}$$

Semi smooth Newton-Raphson algorithm (see [Popp et al.])

$$C_j(\mathbf{t}_j, \mathbf{d}) \;=\; (z_n)_j - \max(0, (z_n)_j - c_n\, \tilde{g}_j), \quad c_n > 0$$

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$$\begin{aligned} \tilde{g}_j &\geq 0 \\ (z_n)_j &\geq 0 \\ (z_n)_j \, \tilde{g}_j &= 0 \end{aligned}$$

Semi smooth Newton-Raphson algorithm (see [Popp et al.])

$$\begin{aligned} C_j(\mathbf{t}_j, \mathbf{d}) &= (z_n)_j - \max(0, (z_n)_j - c_n \, \tilde{g}_j), \quad c_n > 0 \\ C_j(\mathbf{t}_j, \mathbf{d}) &= 0 \end{aligned}$$

After **each** iteration in Newton-Raphson check with new $\tilde{g}_j$, $(z_n)_j$

- Active node: $(z_n)_j - c_n \, \tilde{g}_j > 0$
- Inactive node: $(z_n)_j - c_n \, \tilde{g}_j \leq 0$

Conclusion

Already implemented variants

Penalty	Mortar $\mathbf{n}$	Concentrated $\int$	Fixed Point
Lagrange	Mortar $\mathbf{n}$	Concentrated $\int$	Fixed Point
Lagrange	Average $\mathbf{n}$	Concentrated $\int$	Fixed Point

Conclusion

Already implemented variants and planned variant

Penalty	Mortar $\mathbf{n}$	Concentrated $\int$	Fixed Point
Lagrange	Mortar $\mathbf{n}$	Concentrated $\int$	Fixed Point
Lagrange	Average $\mathbf{n}$	Concentrated $\int$	Fixed Point
Lagrange	Average $\mathbf{n}$	Clipping $\int$	Semi smooth Newton

Conclusion

Already implemented variants and planned variant

Penalty	Mortar $\mathbf{n}$	Concentrated $\int$	Fixed Point
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Lagrange	Average $\mathbf{n}$	Clipping $\int$	Semi smooth Newton

- add friction

Conclusion

Already implemented variants and planned variant

Penalty	Mortar $\mathbf{n}$	Concentrated $\int$	Fixed Point
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Lagrange	Average $\mathbf{n}$	Concentrated $\int$	Fixed Point
Lagrange	Average $\mathbf{n}$	Clipping $\int$	Semi smooth Newton

- add friction
- thermo-mechanical coupled problems

Conclusion

Already implemented variants and planned variant

Penalty	Mortar $\mathbf{n}$	Concentrated $\int$	Fixed Point
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Lagrange	Average $\mathbf{n}$	Clipping $\int$	Semi smooth Newton

- add friction
- thermo-mechanical coupled problems

I am looking forward to answer your questions!

Michael Hammer